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# MICROSATELLITE IN LEO TO MEO ORBITS: MISSION ANALYSIS AND CONTROL STABILITY 

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#### Abstract

The aim of this study is to conduct a mission analysis for a microsatellite with an undefined payload, capable to carry out different missions in orbits with altitudes from Low Earth Orbit (LEO) to Medium Earth Orbit (MEO) and inclinations from zero to ninety degrees. Our goal is to determine proper countermeasures to major factors influencing operational life and platform attitude in order to guarantee operability through a five years mission. Therefore we will conduct a study of the space environment spanning from four-hundred to twelve-hundred kilometres, identifying the worst case for each factor and determining the intensity of its effect. We will take into account the corrosion due to the impacts with atomic oxygen particles, manmade debris and meteoroids and the torques due to aerodynamic drag, solar radiation pressure, geomagnetic field and gravity gradient. A proper countermeasure or the characteristics of a proper actuator will be outlined for each effect. The environmental analysis has been conducted using the world-wide-web based SPace Environment and Information System (SPENVIS) ${ }^{6}$ developed by the European Space Agency (ESA). This study has been done as a graduation thesis for the author's bachelor degree in aerospace engineering.


## INTRODUCTION

Missions employing microsatellites are raising a steadily growing interest in the scientific community due to their flexibility, contained costs and ability to be designed by relatively small groups of people. Furthermore, microsatellites find a large number of applications in Earth sciences and they can be used as a test bed for new materials and technologies. For all these reasons they represent an attractive solution to many research institutes and universities. Until now, the employment of microsatellites has been somewhat restrained by high launch costs. However, the development of new launchers designed to contain such costs
will favour the expansion of this business which will probably become widely spread in the near future ${ }^{1}$.

## A. PLATFORM DESCRIPTION

## A.1. Geometry

The satellite we are considering for our analysis has a parallelepipedon structure measuring 46.8 centimetres in length and width and 68.0 centimetres in height. It is mostly made of composites, carbon and aluminium and has a total mass of 55 kilograms.

A


Fig. 1: Satellite geometry ${ }^{2}$
It is powered by four solar arrays which are not taken into account in the estimation of the perturbing torques.

## A.2. Configuration

The items mounted on this platform are:

1) One Reaction wheel weighing 5.9 kilograms;
2) Three magnetic torquers (one each axis) weighing 1.6 kilograms each;
3) Two battery packs weighing 1.14 kilograms each;
4) Two on board computers weighing 1.2 and 0.9 kilograms respectively;
5) The mission payload weighing 11.6 kilograms ${ }^{2}$.
This yields a total non-structural mass of 26.68 kilograms and a structural mass of 28.32 kilograms.

## A.3. Mission

The mission and consequently the nature of the payload are not defined. However, our satellite is required to be able to operate for a maximum of five years in orbits ranging from 400 Km to 1200 Km with inclinations ranging from 0 to 90 degrees. We will consider the years spanning from January 2005 to December 2009 throughout this paper. The orbits are supposed circular for convenience. We must keep this in mind in dimensioning the protections and actuators that will guarantee the satellite proper functioning throughout the mission.

## A.4. Centre of Mass

Knowing the position of the centre of mass of a given body it is essential when evaluating the magnitude of a torque imparted by a certain
force acting on the body. In general we can write:

$$
\begin{equation*}
T=F \times b \tag{1}
\end{equation*}
$$

where T is the torque, F the force acting on the body and $b$ the distance between the centre of mass and the point on which acts the force F . We must therefore estimate the position of the centre of mass of our satellite in a certain reference frame.
Since the mass distribution inside the satellite case is not constant but is composed of different elements, we proceed in the following manner: the elements listed in the section A.2. are represented as boxes of constant mass density, each weighing like the element it represents. The structural mass is considered distributed equally on the satellite case. We choose a reference frame as in Figure 2. The position of the centre of mass is given by:

$$
\begin{equation*}
\vec{x}_{c}=\sum_{i=1}^{n} \frac{m_{i} \vec{a}_{i}}{M} \tag{2}
\end{equation*}
$$

where the vector quantity $\mathbf{x}_{\mathbf{c}}=(\mathrm{x}, \mathrm{y}, \mathrm{z})$ represents the position of the centre of mass in the chosen reference axis frame, $m_{i}$ is the mass of the element $\mathrm{i}, \mathbf{a}_{\mathrm{i}}=(\mathrm{x}, \mathrm{y}, \mathrm{z})$ is the position of the centre of mass of the element $i$, relative to the chosen reference frame. The mass of each element is considered to be shrunk in its centre of mass.


Fig. 2: Distribution of masses inside the satellite body.
Thus, knowing the dimensions of each element, we can calculate $\mathbf{x}_{\mathbf{C M}}=\mathbf{( 2 2 , 0 4} \mathbf{~ c m} ; \mathbf{2 4 , 6 6 c m}$; $33,14 \mathrm{~cm}$ ).

|  | Element | $\mathrm{m} \mathrm{(kg)}$ |
| :---: | :---: | :---: |
| 1 | Reaction Wheel | 5,9 |
| 2 | MagneticTorquer | 1,6 |
| 3 | Battery Pack | 1,14 |
|  | On Board |  |
| 4 | Computer 1 | 1,2 |
|  | On Board |  |
| 5 | Computer 2 | 0,9 |
| 6 | Payload | 11,6 |

Table 1: Legend for Fig. 2

## B. ATOMIC OXYGEN CORROSION

A satellite moving in Earth orbit is subjected to impacts by ionized particles such as atomic oxygen or hydrogen. This is a consequence of the interaction between the upper atmosphere and solar radiation. Over an extended period of time the friction between such particles and the outer layer of the satellite will corrode the latter. Therefore it is necessary to protect the satellite from corrosion effects in order to ensure its survivability. We will consider only the effect of corrosion due to atomic oxygen.

## B.1. Insulation

Before analysing the effects of corrosion, we need to characterise the kind of material that will constitute the outer layer of our satellite. Since every satellite uses a wide variety of electrical components, it is necessary to shield them from electrical interferences and radiations. Their magnitude largely varies in relation to solar activity; however for a satellite in Low Earth Orbit it is generally sufficient to cover the satellite body with a $50 \mu \mathrm{~m}$ film of Kapton $\mathbb{R}^{3}$. We will now discuss how to determine the width of a layer of Kapton ${ }^{\circledR}$ that will guarantee both the electrical insulation and the protection from atomic oxygen corrosion over a period of five years.

## B.2. Solar and Geomagnetic Indices Forecast

The corrosion due to atomic oxygen is directly proportional to the density of such element. This is inversely proportional to altitude, being higher the closer we get to Earth's atmosphere; furthermore, due to the oblateness of Earth, it is higher at the equator than at the poles. Given the mission parameters in section A.3, this effect is
maximum along an equatorial orbit at an altitude of 400 km . We will assume this parameters in solving the problem.
Having fixed the orbit parameters, the density of atomic oxygen depends on the solar activity (higher when the solar activity is higher) and on the intensity of the Earth geomagnetic field.
The magnitude of radiation coming from the sun is recorded by means of indices such as the F10.7 index, which represents the radio flux in the 10.7 cm bandwidth. The mean value of this index is employed in the model used to calculate the total corrosion. Therefore it is necessary to make forecasts for it in the time span from 2005 to 2009.
At this moment, a forecast of the mean value for the F10.7 index through 2008 is available on the online database of the National Oceanic and Atmospheric Administration (NOAA) ${ }^{4}$, which is reported in Figure 3.


Fig. 3: Forecasts for the F10.7 index.
Source: $\mathrm{NOAA}^{4}$


Fig. 4: Monthly average sunspot number history. Source: NASA

Comparing Figure 3 with the recorded trend in past decades reported in Figure 4, we can extrapolate the trend for 2009.
Thus we can obtain the monthly forecasts for the average F10.7 number shown in Table 2.

|  | $\mathbf{2 0 0 5}$ | $\mathbf{2 0 0 6}$ | $\mathbf{2 0 0 7}$ | $\mathbf{2 0 0 8}$ | $\mathbf{2 0 0 9}$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| January | 80 | 72 | 70 | 82 | 100 |
| February | 79 | 71 | 70 | 84 | 102 |
| March | 78 | 71 | 72 | 86 | 104 |
| April | 77 | 70 | 72 | 88 | 106 |
| May | 77 | 70 | 74 | 90 | 108 |
| June | 76 | 70 | 75 | 90 | 110 |
| July | 76 | 70 | 76 | 92 | 112 |
| August | 75 | 70 | 77 | 94 | 114 |
| September | 74 | 70 | 78 | 96 | 116 |
| October | 73 | 70 | 79 | 96 | 118 |
| November | 73 | 68 | 80 | 98 | 120 |
| December | 72 | 68 | 81 | 100 | 120 |

Table 2: Predicted monthly average F10.7 index
The intensity of the Geomagnetic activity is represented by the Planetary Index Ap. This number is mostly random and there are no forecasts available for it. However, analyzing the trend of recorded values shown in Figure 5, we obtain that, for our analysis, a mean value of 15 can be used.


Fig. 5: Recorded values for the Planetary Index Ap. Source: $\mathrm{NOAA}^{4}$

## B.3. Total Corrosion

We used the world-wide-web based SPace Environment Information System (SPENVIS) ${ }^{6}$ developed by the European Space Agency (ESA) to calculate the width of a generical Kapton®
film (erosion yield: $3.00 \cdot 10^{-24} \mathrm{~cm}^{3}$ ) eroded during the five years considered in section A.3. Inserting the values of the F10.7 and Ap indices found in section B.2, we obtained the results shown in Table 3.

|  | Front Erosion Depth (cm) |
| :---: | :---: |
| 2005 | $3,79 \cdot 10^{-3}$ |
| 2006 | $3,24 \cdot 10^{-3}$ |
| 2007 | $3,56 \cdot 10^{-3}$ |
| 2008 | $5,08 \cdot 10^{-3}$ |
| 2009 | $7,32 \cdot 10^{-3}$ |
| Mission Total <br> $(\mathrm{cm})$ | $\mathbf{2 , 2 3 \cdot 1 0}$ |

Table 3: Total erosion due to atomic oxygen.
Source: SPENVIS ${ }^{6}$

This result in combination with the statements in section B.1, yield to a total Kapton ${ }^{\circledR}$ cover width of $\mathbf{2 . 3 0 \cdot 1 0 ^ { - 2 }} \mathbf{~ c m}$.

## C. IMPACTS WITH METEOROIDS AND DEBRIS

In the previous sections we discussed how to determine the width of a Kapton ${ }^{\circledR}$ film that will guarantee both electrical insulation and protection from atomic oxygen erosion for five years. In this section we will determine is if the result found is also sufficient for protection against impacts of micrometeoroids or small particles of manmade debris. To do that, we shall first determine what kind of particles can be expected to be found between 400 and 1200 Km .

## C.1. Micrometeoroids

The flux of micrometeoroids is sensibly constant with altitudes and with time (at least for the duration and altitude range of the mission).
A flux versus mass of meteoroids particles plot is shown in Figure 6. All particles are considered to be spherical with a constant mass density. We can see from Figure 6 that the probability of an impact with meteoroids having a mass ranging from $10^{-17}$ to $10^{-5}$ grams is much higher (at least about seven orders of magnitude) than that of an impact with particles of masses from $10^{-4}$ up to 1 gram.


Fig. 6: Meteoroids flux for an altitude of 400 Km .
Source: SPENVIS ${ }^{6}$

We shall therefore verify that a film of Kapton ${ }^{\circledR}$ has the necessary impact strength to resist impacts with a sphere of $10^{-5}$ grams. Considering for convenience impacts with velocity vectors normal to the satellite surface, we have:

$$
\begin{equation*}
\frac{1}{2} m v^{2} \leq I \tag{3}
\end{equation*}
$$

where the left member of the inequation represents the particle kinetic energy ( m and v are the particle mass and velocity respectively) and the right member represents the impact strength of the material exposed to the particle impacts. Pneumatic impact tests show that for a $25 \mu$ film of Kapton® at $23^{\circ} \mathrm{C}^{3}$

$$
\begin{equation*}
I=0.78 \mathrm{Nm} \tag{4}
\end{equation*}
$$

The velocity for a circular orbit of 400 Km of altitude is:

$$
\begin{equation*}
v=\sqrt{\frac{\mu}{\left(R_{E}+h\right)}}=7.67 \mathrm{~km} / \mathrm{s} \tag{5}
\end{equation*}
$$

being $\mu=3.986012 \cdot 10^{5} \mathrm{Km}^{3} / \mathrm{s}^{2}$ the Earth Gravitational Parameter, $\mathrm{R}_{\mathrm{E}}=6378.145 \mathrm{Km}$ the Mean Equatorial Radius and $h$ the altitude of the circular orbit. Therefore, the velocity of a particle relative to the spacecraft is in the order of magnitude of $10 \mathrm{Km} / \mathrm{s}$. For a particle having a mass of $10^{-5}$ grams from inequation (3) we obtain:

$$
\begin{equation*}
0.5 J<0.78 J \tag{6}
\end{equation*}
$$

which tells us that a layer of $25 \mu$ of Kapton ${ }^{\circledR}$ constitutes a sufficient protection against impacts with particles with a mass of $10^{-5}$ grams or less.

## C.2. Manmade Debris

The flux of debris is sensibly constant with altitude (at least for the altitude range considered for the mission); however it increases with time as more and more manned and unmanned missions are launched. Therefore, for the considered time span, we will have the worst flux in the year 2009. A plot of the particle flux versus diameter is shown in Figure 7.


Fig. 7: Debris flux for the year 2009 for an altitude of 400 Km. Source: SPENVIS ${ }^{6}$

As before, all debris particles are assumed to be spherical with a constant mass density. The plot in Figure 7 shows that the probability of an impact with debris particles having a diameter ranging from $10^{-3}$ to $10^{-2}$ centimetres is considerably higher (at least about two orders of magnitude) than that of an impact with particles of diameters from $10^{-1}$ centimetres up to 1 metre. As before, we shall verify that a film of Kapton® has the necessary impact strength to resist impacts with a sphere of $10^{-2} \mathrm{~cm}$ in diameter and compare it with the value in equation (4). If we suppose it to be made of aluminium (the most common element employed in aerospace applications) with a density $\rho=2.7 \mathrm{~g} / \mathrm{cm}^{3}$, we obtain that a sphere of $10^{-2} \mathrm{~cm}$ in diameter has a mass of about $1.42 \cdot 10^{-6}$ grams.
Repeating the procedure described in section C.1, from equations (4) and (5) we obtain:

$$
\begin{equation*}
7.1 \cdot 10^{-2} J<0.78 J \tag{7}
\end{equation*}
$$

which tells us that a layer of $25 \mu$ of Kapton ${ }^{\circledR}$ constitutes a sufficient protection against impacts with particles having a mass of $10^{-6}$ grams or less.
Hence, from the results obtained in sections C. 1 and C.2, we can conclude that the cover film
width calculated in section B. 3 should be enough to shield the spacecraft against impacts with particles with mass equal or less than $10^{-6}$ grams. For higher masses the particle might penetrate the outer layer.

## D. ATTITUDE CONTROL


y (Pitch Axis)
Fig. 8: Attitude Control Angles.
The orientation in space (attitude) of a satellite in Earth orbit is perturbed by torques that make it spin about a reference frame centred in its centre of mass. Such torques are induced mainly by the following perturbing forces: aerodynamic drag, solar radiation pressure, gravity gradient and magnetic interactions. Since the satellite is designed to function in a given orientation, it is necessary to determine a way to counteract such effects. In the following sections we will determine the magnitude of the effect produced by each force about the reference frame shown in Figure 8. The axis orientation is parallel to that of the axis in Figure 2, but they are centred in the satellite centre of mass found in section A.4. In the last section we will outline the general characteristics of the actuators needed to keep the satellite in the desired orientation.

## D.1. Aerodynamic Drag

A body moving in Low Earth Orbit is subjected to a force due to aerodynamic drag. Although above 100 Km the atmosphere is highly rarefied and constituted primary by lone atoms, such
presence is enough to exert an appreciable force on a body. If we suppose our satellite to be moving along the positive x axis (as in Figure 8) and the flow of gas particles to be perpendicular to the plane defined by the $y$ and $z$ axis, we can write equation:

$$
\begin{equation*}
D=\frac{1}{2} \rho V^{2} S C_{D} \tag{8}
\end{equation*}
$$

where D is the aerodynamic drag, $\rho$ is the gas density, V is the flow velocity, S the surface exposed to the flow current and $C_{D}$ the drag coefficient. Since the gas density is inversely proportional to altitude, it follows that the highest drag value will be found in the lowest orbit. We will therefore assume an altitude of 400 Km ; here the value for $\rho$ is about $2.8031 \cdot 10^{-12}$ $\mathrm{Kg} / \mathrm{m}^{3}$ (see reference 7). The velocity for a circular orbit at this altitude is reported in equation (5); the surface $S$ is $0.337 \mathrm{~m}^{2}$ and the drag coefficient for a plane sheet in a hypersonic flow is 2 . Hence from (8) we have:

$$
\begin{equation*}
D=5.56 \cdot 10^{-5} N \tag{9}
\end{equation*}
$$

The centre of pressure is located at the barycentre of the satellite face parallel to the xy plane, thus the distance between the centre of pressure and the centre of mass is $24.79 \mathrm{~cm}^{*}$. Using equation (1) we can calculate the torque $\mathrm{T}_{\mathrm{A}}$ due to aerodynamic drag:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{A}}=\mathrm{Db}=1.378 \cdot 10^{-5} \mathrm{Nm} \tag{10}
\end{equation*}
$$

where the torque arm $\mathbf{b}$ is represented by the distance CP-CM. This mainly acts about the Pitch Axis.

## D.2. Solar Radiation Pressure

The solar radiation pressure is due to both solar radiation and solar wind. The solar radiation comprises all the electromagnetic waves radiated by the Sun, while the solar wind mainly consists of ionized particles such as nuclei and electrons. This pressure is proportional to the momentum flux of the radiation; since the momentum of the

[^0]radiation alone is much greater than that of the solar wind (by a factor from 100 to 1000), the latter can be ignored. The momentum flux largely depends on the solar activity and on the position of the Earth in its orbit around the Sun; however, for our purposes, we can assume a mean momentum flux ${ }^{8}$ : $\mathrm{P}=4.5 \cdot 10^{-6} \mathrm{Kg} / \mathrm{m} \cdot \mathrm{sec}^{-2}$. The force due to the solar radiation pressure $S_{p}$ is proportional to P according to equation:
\[

$$
\begin{equation*}
S_{P}=P A k \tag{11}
\end{equation*}
$$

\]

where A is the area of the surface on which the pressure acts and k is a coefficient that depends on the absorption characteristics of the satellite. We will assume that the material covering the satellite acts as a black body; thus $\mathrm{k}=1$. The force $S_{p}$ varies along an orbit, in fact it is higher when the satellite is directly exposed to the Sun. The worst situation for our satellite would be to be placed in a polar orbit, since it would always be exposed to the sun. Under such hypothesis, we can assume the force $S_{p}$ exerted on the body to be fairly constant and equal to:

$$
\begin{equation*}
S_{P}=1,517 \cdot 10^{-6} \mathrm{~N} \tag{12}
\end{equation*}
$$

If we suppose the satellite to be moving on a polar orbit with its velocity vector oriented as in Figure 8, the radiation flux from the Sun would come from the direction of the Pitch Axis y. If we also suppose it to be parallel to this axis, since in this case the effect produced on the spacecraft would be the highest one, the distance between the centre of mass a and the centre of pressure (the torque arm) is about $24.71 \mathrm{~cm}^{\dagger}$. Thus, the torque $T_{s}$ produced by the force $S_{p}$ is, according to equation (1):

$$
\begin{equation*}
\mathrm{T}_{\mathrm{S}}=\mathrm{S}_{\mathrm{P}} \mathrm{a}=3.747 \cdot 10^{-7} \mathrm{Nm} \tag{13}
\end{equation*}
$$

which acts about the Roll and Yaw axis.

## D.3. Gravity Gradient

Since a spacecraft is a three dimensional body, each of its points has a different distance from the Earth centre of mass. This difference, even if small, produces a different gravitational pull on

[^1]each point in the body. This effect is called Gravity Gradient and it produces a torque that tends to make the satellite spin so that the axis of least inertia is pointed towards the Earth centre of mass. This could be a perturbing force, unless it is used as a passive attitude control system; it can be done because the magnitude of the effect of the gravity gradient is predictable and depends only on the distribution of the masses inside the satellite, on its geometry and altitude.
This is the case. The geometry and distribution of the masses inside our satellite are such that the gravity gradient acts so to maintain the nominal attitude.


Fig. 9: Stabilizing effect of the gravity gradient
The estimation of the value of the torque produced by the gravity gradient presents some difficulties and it is beyond the aim of this paper. It is sufficient to say that, under the hypothesis made, during the mission (after the nominal attitude is acquired) it does not require counteracting measures.

## D.4. Geomagnetic Field

The Earth magnetic field interacts with the magnetic field generated by electric currents flowing through the satellite. Normally, this would produce perturbing forces and torques; however, as for the gravity gradient, the predictability of such effect allows us to use it as an active attitude control system by introducing proper coils called Magnetic Torquers. The magnetic field produced by one coil interacts with the geomagnetic field producing a torque $\mathrm{T}_{\mathrm{m}}$ given by equation:

$$
\begin{equation*}
T_{m}=m \times B \tag{14}
\end{equation*}
$$

where m is the coil magnetic dipole moment and B the Earth magnetic field vector. Positioning one actuator along each axis, the total torque produced counteracts those given by the aerodynamic drag and the solar radiation pressure. In order to dimension a proper actuator, we need to know the intensity of B , which is shown in Figure 9. We have considered an altitude of 400 Km and the Earth magnetic field to be constant with time, which is a fairly accurate assessment over a period of five years. We see from Figure 10 that the value of B ranges from a maximum of about 0.35 G to a minimum of about $0.22 \mathrm{G}\left(3.5 \cdot 10^{-5} \mathrm{~T}\right.$ to $\left.2.2 \cdot 10^{-5} \mathrm{~T}\right)$.


Fig. 10: Predicted Earth magnetic field vector along an orbit 400 Km in altitude for the year 2005. Source: SPENVIS ${ }^{6}$

## D.5. Characteristics of the Actuators

In this section we will assume an orbit altitude of 400 Km since we have shown that most of the environmental effect depending on altitude are worst here.

A reaction wheel is essentially a rotor spun by an electric motor. When a torque is applied to the wheel to speed it up, it produces a reacting torque on the spacecraft body. Since it has a fixed axis, one reaction wheel can act about only one axis. Our satellite only wheel produces a torque about the pitch axis. It must, therefore, be able to balance out the torque produced by the aerodynamic drag (section D.1). Consequently, the wheel must be able to produce a torque of
$1.378 \cdot 10^{-5} \mathrm{Nm}$. In order to produce a constant torque, however, the electric motor must constantly increase the angular momentum of the wheel. This means that the rotor must have a constant acceleration. When it reaches its maximum speed (saturation) it will need to be slowed down. This action will cause a torque on the satellite body that constitutes an attitude perturbation. We can estimate the time it will take the reaction wheel to reach saturation using equation:

$$
\begin{equation*}
\Gamma_{\max }=T_{W} \Delta t \tag{15}
\end{equation*}
$$

where $\Gamma_{\text {max }}$ is the maximum angular momentum of the wheel, $\mathrm{T}_{\mathrm{v}}$ is the torque produced by the wheel and $\Delta \mathrm{t}$ is the interval of time. If we assume the wheel to have a maximum angular momentum of 4 Nms (such actuators are commonly found on the market) and $\mathrm{T}_{\mathrm{W}}=\mathrm{T}_{\mathrm{A}}$, equation (15) yields a saturation time of about 80 hours (or once every 52 orbits). In general, the reaction wheels available on the market and satisfying this requirement have a maximum torque of at least 12 mNm (sometimes even 20 mNm ), which largely meets our needs.

Magnetic torquers have already been briefly introduced in section D.4. From equation (14) we can deduce that the torque intensity is given by:

$$
\begin{equation*}
\left|T_{m}\right|=m B \operatorname{sen} \theta \tag{16}
\end{equation*}
$$

where $\theta$ is the angle that B forms with m . It is easily seen that a magnetic torquer does not produce any effect if its axis is lined up with the earth magnetic field. On the other hand, the maximum torque produced by one torquer is:

$$
\begin{equation*}
\left|T_{m}\right|=m B \tag{17}
\end{equation*}
$$

if its axis is normal to B. Since every torquer acts only on the two axis normal to its own, at any given moment we could indifferently have two or one (this is the case expressed by equation (17)) actuators acting on one axis. Hence, we require one torquer to have a maximum torque sufficient to counteract all the perturbing torques acting on a given axis.
One of the actuators acting on the roll and yaw axis must balance out the effect produced by the solar radiation pressure (section D.2), thus it must produce a torque of $3.747 \cdot 10^{-7} \mathrm{Nm}$. It must therefore have a maximum dipole moment given by equation:

$$
\begin{equation*}
m=\frac{T_{S}}{B_{\min }} \tag{18}
\end{equation*}
$$

where $B_{\text {min }}$ is the minimum predicted value of $B$ for a 400 Km high orbit. From section D. 4 we have that $\mathrm{B}_{\min }=2.2 \cdot 10^{-5} \mathrm{~T}$. From equation (18) we have therefore: $\mathrm{m}=0.017 \mathrm{Am}^{2}$.
When the reaction wheel is slowed down, one of the torquers acting on the pitch axis must be able to balance out the torque produced by the atmospheric drag and to compensate for the torque produced by the wheel itself. When it reaches saturation, it does not accelerate (its velocity is constant and equal to the maximum velocity) and does not produce a torque anymore. If we assume a de-saturation time of two orbits (or about three hours) as acceptable, from equation (15) we have:

$$
\begin{equation*}
T_{W}=\frac{\Gamma_{\max }}{\Delta t} \tag{19}
\end{equation*}
$$

where $\mathrm{T}_{\mathrm{W}}$ is now the torque impressed on the wheel to slow it to an alt. Equation (19) yields to a torque of $3.60 \cdot 10^{-4} \mathrm{Nm}$. Thus we have:

$$
\begin{equation*}
m=\frac{T_{A}+T_{W}}{B_{\min }} \tag{20}
\end{equation*}
$$

Equation (20) yields a dipole moment $\mathrm{m}=17 \mathrm{Am}^{2}$. Some of the smallest torquers on the market have a maximum linear dipole moment of $30 \mathrm{Am}^{2}$, which fits our needs. Such dipole moment is also sufficient to slow the satellite down while de-tumbling. The results of this section are summarized in the possible actuators configuration shown in Table 4 below.

|  | Quantity | Max <br> Wheel <br> Momentum | Max <br> Wheel <br> Torque | Mass |
| :---: | :---: | :---: | :---: | :---: |
| Reaction <br> Wheel | 1 <br> (Pitch <br> Axis) | 4 Nms | 12 mNm | 5.9 <br> Kg |


|  | Quantity | Linear <br> Dipole <br> Moment | Saturation <br> Moment | Mass |
| :---: | :---: | :---: | :---: | :---: |
|  | 3 <br> $($ One | $30 \mathrm{Am}^{2}$ | $40 \mathrm{Am}^{2}$ | 1.6 |
| Kg |  |  |  |  |
| Magnetic <br> Torquers | (ach <br> Axis) |  |  |  |

Tab. 4: Possible configuration of the satellite actuators

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[^0]:    * In the reference frame of Fig. 2 the centre of pressure is $\mathrm{CP}=(46.8 \mathrm{~cm} ; 23.4 \mathrm{~cm} ; 34 \mathrm{~cm})$ and the centre of mass is $\mathrm{CM}=(22,04 \mathrm{~cm} ; 24,66 \mathrm{~cm}$; $33,14 \mathrm{~cm}$ ); the distance CP-CM is:
    $\sqrt{\left(x_{C P}-x_{C M}\right)^{2}+\left(y_{C P}-y_{C M}\right)^{2}+\left(z_{C P}-z_{C M}\right)^{2}}=24.79 \mathrm{~cm}$

[^1]:    ${ }^{\dagger}$ In the reference frame of Fig. 2 the centre of pressure is $\mathrm{CP}=(23.4 \mathrm{~cm} ; 0 \mathrm{~cm} ; 34 \mathrm{~cm})$ and the centre of mass is $C M=(22,04 \mathrm{~cm} ; 24,66 \mathrm{~cm} ; 33,14 \mathrm{~cm})$; the distance CP-CM is:

    $$
    \sqrt{\left(x_{C P}-x_{C M}\right)^{2}+\left(y_{C P}-y_{C M}\right)^{2}+\left(z_{C P}-z_{C M}\right)^{2}}=24.71 \mathrm{~cm}
    $$

